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MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Prologue

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Prologue

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[Plate 1]

The distance between a laser transmitter on Earth and corner reflectors in space can now be measured with a precision of a few centimetres. The main theme of the meeting is the exploitation of such measurements between stations on Earth and reflectors embedded in artificial satellites or placed on the Moon. This Prologue prepares the way for the more specialized papers by reviewing the advances already made in geodesy and geophysics using less accurate observations of artificial satellites, and by briefly outlining previous lunar dynamical studies.

O for a Muse of fire, that would ascend
The brightest heaven of invention....

These lines, the opening of the Prologue of Shakespeare's *King Henry V*, seem an appropriate motto for this meeting, which sports a ruby laser as its *Muse of fire*, emitting pulses which *ascend* into *heaven* as the *brightest* light that human *invention* has yet devised.

The main theme of the meeting is the exploitation of the very accurate methods now becoming available for measuring the distance between a laser transmitter on Earth and corner reflectors in space: these reflectors may either be embedded in artificial Earth satellites or be placed on the Moon. Accuracies of order 10 cm are being achieved in such measurements, and a precision of order 1 cm is being seriously discussed as a possibility in the future.

The individual papers in this volume cover a wide range of subjects and scientific disciplines. First, the present and future instrumental techniques are discussed, not only for lasers, but also for other complementary methods of high precision, such as very-long-baseline interferometry, satellite-to-satellite tracking and altimetry from satellites. Next, the extensive results already obtained, mainly from laser tracking of satellites, are presented: these include improved determinations of the Earth's gravitational field and geoid shape, the Earth's polar motion, ocean tides and solid Earth tides, and the linking of variations in the Earth's rotation rate with atmospheric winds. Then the discussion turns to the likely future advances from the use of space ranging. Already the lunar laser ranging has demanded great improvements in lunar orbital theory and lunar libration theory; future measurements of distance to the reflectors on the Moon, interpreted by improved theory, should greatly advance knowledge of lunar dynamics, including assessment of relativity effects, and can give new information on the Earth's rotation rate and polar motion. Ranging to Earth satellites using new and more accurate lasers with satellites of improved design, such as Lageos, promises direct measurements of the relative movements between tectonic plates, and improvements in all the geophysical measurements previously mentioned; many new geophysical studies should also become possible, in regions running from the deep interior of the Earth to the upper atmosphere, and including accurate measurements of the sea-surface profile from laser ranging combined with altimetry.

My task in this Prologue is to lead up to the main theme of the meeting by briefly reviewing previous progress in studies utilizing observations of artificial satellites and the Moon.

With artificial satellites the progress has been in the last 20 years, using the obvious ancestors of observations to be discussed in later papers in this volume. These pre-laser observations of satellites are accurate to 5–10 m at best and, backed up by observations of 200 m accuracy, have led to major advances in geophysics, which I shall quickly review. Further researches along these lines and many new ones will be possible with the observations 100 times more accurate now being made.

With the lunar studies the prehistory is rather different, and goes back to the time of Newton. The laser measurements of distances to reflectors on the Moon, being much more accurate than previous techniques, open a new area of lunar research, requiring new approaches and in particular much improved orbital and libration theory.

Previous researches using observations of artificial Earth satellites

The obvious predecessor of the laser in Earth-satellite tracking is the large camera of Schmidt type, of which the Baker–Nunn camera of the Smithsonian Astrophysical Observatory (Hayes 1962–5) is the best-known example. The Baker–Nunn camera was designed in the mid-1950s for photography of satellites illuminated by the Sun against a dark sky. The aperture of the camera is 500 mm and the field of view 30° × 5°. A three-axis gimbal system allows the camera to follow a satellite across the sky, thereby enhancing the brightness of the image. The positions of the satellite relative to the stars at known times are recorded on photographic film, and can be measured accurate to about 2″, equivalent to 10 m in position at a distance of 1000 km. Satellites as faint as magnitude 12 can be recorded: a satellite 30 cm in diameter can therefore be tracked out to 2000 km range. Over a million observations of satellites have been made by the Baker–Nunn cameras, and the system remains in operation.

The most accurate of the satellite cameras is the Hewitt camera, owned and operated by the Ordnance Survey at Malvern in Britain (figure 1, plate 1). The Hewitt camera has an aperture of 600 mm, slightly larger than the Baker–Nunn, but the photographs are taken with the camera stationary, with the result that it cannot record such faint satellites: the limiting magnitude depends on the angular rate of travel of the satellite across the sky, but is about magnitude 8 for a typical satellite. The design of the camera (Hewitt 1965) incorporates a field-flattening lens near the focus, so that the stars and satellite can be recorded on a glass photographic plate. The accuracy is about 1″, equivalent to 5 m for a satellite at a distance of 1000 km.

These, and many other cameras, can be regarded as the élite among the traditional tracking methods, and their observations are supported by radar, visual and radio interferometer methods, which have accuracies of 1 or 2′, equivalent to 150 or 300 m positional accuracy at a range of 500 km. So, for these traditional pre-laser tracking methods, it is fair to say that the observations are of about 200 m accuracy, with camera observations of 5–10 m as the 'cutting edge' of the technique, as it were.

Another important method of tracking Earth satellites is by radio Doppler techniques, which in the refined U.S. Navy Navigation Satellite System give about 5 m accuracy, similar to the cameras. Doppler has also been the standard method for tracking satellites in orbit about the Moon.

Although the new laser techniques are far more accurate, the traditional techniques will retain their value, because there are now more than 3800 objects in space, catalogued and tracked by

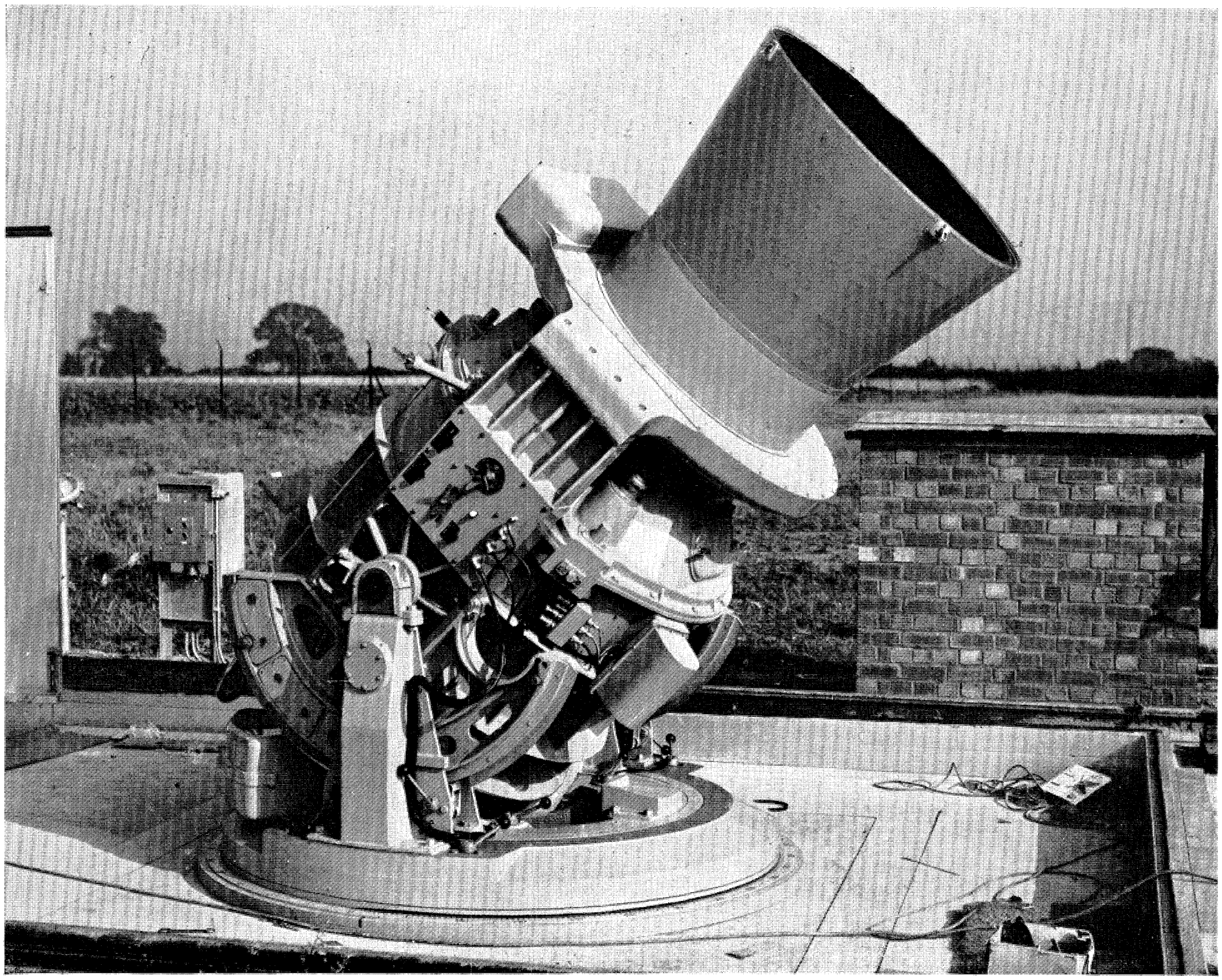

FIGURE 1. The 600 mm Hewitt camera operated by the Ordnance Survey at Malvern.

(*Facing p.* 422)

the U.S. Air Force radar network; and only 12 of these 3800 satellites are fitted with the corner reflectors necessary for laser tracking. As a very rough guide, about 150 of the satellites transmit radio signals, and therefore might possibly be tracked by radio methods; about 1500 may come within the grasp of the Hewitt camera, and about 3000 within the grasp of the Baker–Nunn cameras. For geophysical studies based on satellite observations, it is necessary to select satellites in the orbits most suitable for the purpose, and generally these are only likely to be observable by optical and radar methods. So the traditional methods of tracking will remain useful, and indeed the Hewitt camera at Malvern is to be used more intensively in the next few years, after a grant by the Science Research Council in support of optical tracking.

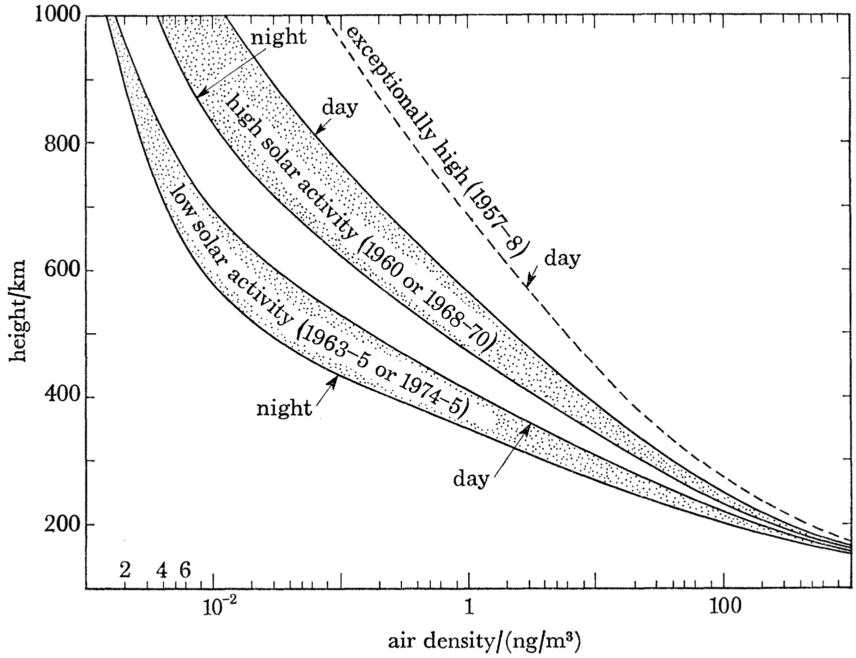

FIGURE 2. Density versus height from 150 to 1000 km for low and high solar activity.

The results that have come from these traditional methods of tracking, either by analysing orbits or by geometrical methods, have been primarily in two separate areas: (*a*) the upper atmosphere and (*b*) geodesy and the Earth's gravitational field. In reviewing the results I shall start with those that can be obtained using relatively inaccurate observations, and then progress towards the more accurate observations.

The upper atmosphere became a prime target for orbit analysts as soon as the satellites started moving through it in 1957, registering its integrated effects by changes in their orbits. A satellite moving in an elliptic orbit suffers the greatest air drag when it comes nearest to the Earth, at perigee; so the effect of air drag is to retard the satellite as it passes through the region near perigee. Consequently, it does not swing out so far on the opposite side of the Earth, and the maximum height is steadily reduced while the perigee height remains almost the same. The orbit contracts and becomes more nearly circular. The rate at which the orbit contracts can be very accurately measured, and gives a measure of the density of the air in the regions near perigee (King-Hele 1964).

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Many thousands of measurements of air density have been made using this method, and the results are summarized in figure 2, which shows the variation of density with height in the region of the atmosphere where the technique is most powerful – between 100 and 1000 km. The air density varies greatly with solar activity, being much greater at sunspot maximum, in 1969, than at sunspot minimum in 1975: the density is 20 times greater at heights near 600 km, where the variation is widest. (At the very high sunspot maximum in 1958, the density at 600 km was 100 times greater than at the subsequent solar minimum.) There are also strong variations between day and night, with the density falling to a minimum value at about 04 h local time, rising to a maximum at about 14 h local time, and then decreasing again. As figure 2 shows, the maximum daytime density may exceed the minimum night-time density by a factor of 6 at 500 km height for low solar activity, or at 800 km for high solar activity. Quite apart from these huge 11-year and daily variations, the density exhibits large changes, by a factor of 2 or sometimes up to 5, in a few days or even a few hours, in response to outbursts of solar activity. Density variations also show a 27-day recurrence tendency, and a semi-annual variation by a factor of up to 2.5 with maxima in April and October, and minima in January and July. There are many other variations, of immense complexity, but the successful separation of these major variations is in itself a tribute to the power of the technique.

Most of the variations in density can be ascribed to heating of the upper atmosphere by solar extreme ultraviolet radiation, and by the influx of particles from the solar wind. The temperature of the upper atmosphere, which varies little with height above 250 km, ranges from 600 K, at sunspot minimum by night, to 1200 K at an average sunspot maximum by day. (For further details, see the COSPAR International Reference Atmosphere (CIRA 1972), Jacchia (1975) or King-Hele (1975).)

Satellite orbits determined from photographic, visual and radar observations may also be analysed to determine zonal wind speeds in the region of the perigee. These techniques will be discussed in a later paper in this volume (p. 555).

Although these upper-atmosphere studies need to be borne in mind to keep a proper perspective, the main field of application of satellite observations relevant to this meeting is what is usually called satellite geodesy. The name covers geometrical geodesy, using space triangulation with camera observations, or trilateration with electronic methods or lasers, and also the determination of the gravitational field from orbital analysis. The gravitational field may appropriately be discussed first, since much can be achieved with relatively inaccurate observations.

Space geodesy began a very long time ago, in the third century B.C. when Eratosthenes of Alexandria measured the circumference of the Earth, and probably came within 1 % of the correct value of just over 40 000 km. He noted that when the Sun was overhead at Aswan, it was 7.2° or one-fiftieth of a circle away from the vertical at Alexandria, which was almost due north; so the Earth's circumference was 50 times the distance from Aswan to Alexandria, and this distance he is said to have calculated by multiplying the average speed of a camel by the time it took on the journey.

Space geodesy did not really come into its own, however, until the launch of artificial satellites in 1957. There was one obvious perturbation which could be analysed with relatively inaccurate observations: the departure of the gravitational field from spherical symmetry, caused by the Earth's flattening, makes the orbital plane of a satellite rotate quite rapidly about the Earth's axis from east to west at a rate of order 5° per day, while maintaining the same inclination to the equator. These effects are about a million times greater for a close satellite than for the Moon.

The rate of rotation of the early Sputnik orbits was accurately measured to give a much improved value of the Earth's flattening in 1958, and the value now established is one part in 298.25, so that the equatorial diameter exceeds the polar diameter by 42.77 km.

Even with relatively inaccurate observations, the Earth's gravitational field can be studied in much more detail, because the longitude-independent, or longitude-averaged, components in the geopotential produce several strong effects on orbits. The longitude-averaged potential $\overline{U}$ at an exterior point distant r from the Earth's centre at latitude ϕ may be written

$$\overline{U} = \frac{\mu}{r}\left\{1 - \sum_{n=2}^{\infty} J_n \left(\frac{R}{r}\right)^n P_n(\sin\phi)\right\}, \tag{1}$$

where μ is the gravitational constant for the Earth ($398\,601\ \mathrm{km^3\,s^{-2}}$), R is the Earth's equatorial radius (6378.14 km), $P_n(\sin\phi)$ is the Legendre polynomial of degree n and argument $\sin\phi$, and the J_n are constant coefficients which have to be determined. The first harmonic ($n = 1$) does not appear because the origin is taken at the Earth's centre of mass.

The J_2 term in equation (1) specifies the flattening, the main departure of the Earth from a sphere, and J_2 is much larger than the later coefficients in the series. The third harmonic (the J_3 term) corresponds to a triangular or pear-shaped Earth; the fourth harmonic is square-shaped, the fifth has five petals, and so on. These shapes are the form of the geoid or sea-level surface that would arise if just that one harmonic (and the main spherical term) existed. The sea-level surface of the real Earth is made up of a combination of numerous harmonics, and the coefficients ought to be evaluated up to a very high degree, although in practice the series is usually truncated and evaluations have rarely been carried above degree 25. The expression of the gravitational field as a series of harmonics is only one of many possible representations, but it has proved to be the most suitable in evaluations using satellite orbits.

In determining the J coefficients, the even harmonics $J_2, J_4, J_6, \ldots$ are found by measuring the rate of rotation of the orbital plane, and also the rate of rotation of the perigee within the orbital plane, for a number of satellites. The perigee movement, which is of order 5° per day, carries the perigee point from the northern to the southern hemisphere within a few months for most satellites. The odd zonal harmonic coefficients $J_3, J_5, J_7, \ldots$ are found by analysing long-period oscillations in several orbital elements, and particularly the oscillation in perigee height, which is generally of order 10 km, perigee usually being further from the Earth's centre when in the southern hemisphere than in the northern. The amplitude varies considerably with the inclination of the satellite, however, and can be as much as 60 km for orbits at inclinations near 63°.

Figure 3 shows the shape which emerges on slicing the Earth through the poles and averaging over all longitudes. This is an equipotential surface of the longitude-averaged potential, with the appropriate centrifugal potential added in; or, in physical terms, the shape of the mean sea-level surface, extended under the land in a logical manner and averaged over all longitudes. Figure 3 shows that if you place yourself, in great discomfort, at sea level at the North Pole, you would be 44 m further from the equator than an equally hardy explorer who burrowed down to sea level at the South Pole. This tendency of about 40 m towards a pear shape in the figure of the Earth gives rise to asymmetries in the geopotential, which cause the perturbations of order 10 km in the perigee height: so the orbital perturbation is about 200 times greater than the terrestrial irregularity causing it. That is the key to the success of the method: observations accurate to 200 m can give geoid profiles accurate to 1 m – the accuracy probably achieved in figure 3, which is based on sets of zonal harmonics obtained by Wagner (1973) and by King-Hele & Cook (1974).

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This pear-shaped polar slice provides an excellent and memorable picture of the Earth's shape averaged over all longitudes. But the variations with longitude also need to be studied. To do so, we have to express the geopotential not just as an infinite series of harmonics dependent on latitude, but as a double infinite series of tesseral harmonics dependent on latitude and longitude.

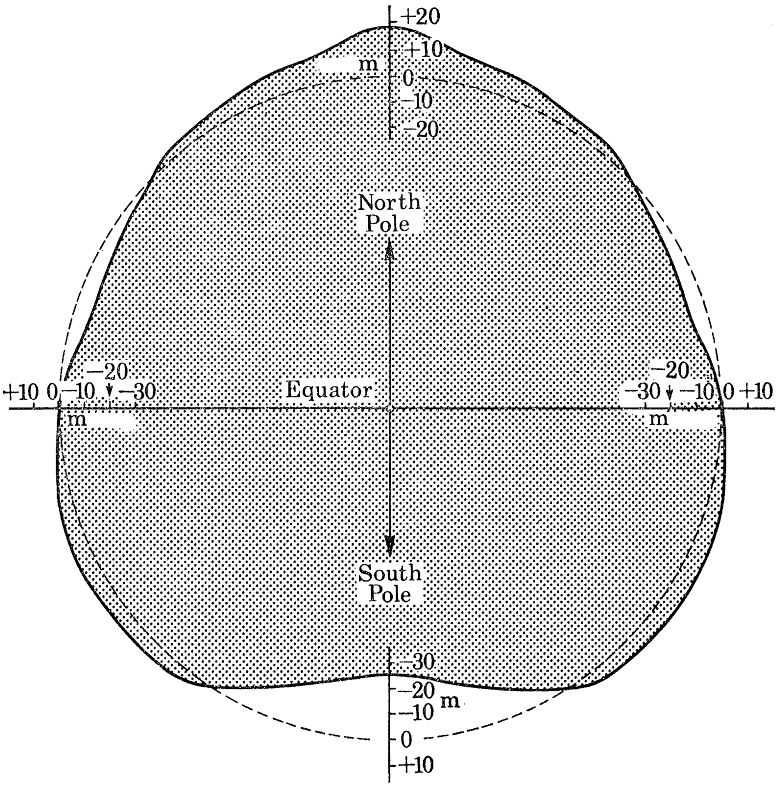

FIGURE 3. Height of the meridional geoid section (solid line) relative to a spheroid of flattening 1/298.25 (broken line), with the even harmonics of Wagner (1973) and the odd harmonics of King-Hele & Cook (1974).

If λ denotes longitude (positive to the east) the geopotential U may be written

$$U = \overline{U} + \frac{\mu}{r} \sum_{l=2}^{\infty} \sum_{m=1}^{l} \left(\frac{R}{r}\right)^l P_l^m(\sin\phi) \{C_{lm}\cos m\lambda + S_{lm}\sin m\lambda\}, \tag{2}$$

where $\overline{U}$ is the longitude-averaged potential of equation (1), $P_l^m(\sin\phi)$ is the associated Legendre function of order m and degree l (with $l \geqslant m$), and the C_{lm} and S_{lm} are constants to be determined. In equation (2) the suffix m may be regarded as specifying the variations in travelling from one meridian to another; while, for fixed m, the suffix l governs the variations in going from one latitude to another. To be more specific, the (l, m) tesseral harmonic, of degree l and order m, has $2m$ zeros (and thus m undulations) as the longitude goes through 360°, and $(l-m)$ zeros as latitude goes through 180°, from pole to pole. Thus the (15, 15) harmonic is like an orange with 15 segments; and the (12, 9) harmonic has pear-shaped meridional sections (because $l-m = 3$), and latitudinal sections with 9 undulations in 360°.

To determine the complete contour map of variations with latitude and longitude, large numbers of accurate observations are needed, and a very large computer. Again, however, there are some orbital effects which can build up as time goes on, so that there are some harmonics which can be evaluated using relatively inaccurate observations. These effects occur with orbits which are resonant with respect to particular harmonics, an orbit being resonant if its orbital period is such that its track over the Earth repeats day after day. For example, 15th-order resonance occurs if the Earth spins through exactly 24° relative to the orbital plane while the satellite is making one revolution, because the Earth will then spin through exactly 360° during 15 revolutions and the track will repeat itself. This repetition leads to a build-up of perturbations due to 15th-order harmonics, and these enhanced effects can be measured quite easily. If a satellite passes slowly through 15th-order resonance as its orbit contracts under the influence of air drag, the change in inclination can be quite large: for example, Cosmos 72 (1965–53B), which passed through resonance during 1972, suffered a decrease in inclination of 0.07°, which is equivalent to a change of 8 km in the maximum latitude attained. Analysis of changes in a number of orbits of this type has given values for geopotential coefficients of order 15 (King-Hele, Walker & Gooding 1975). Similarly, synchronous satellites can give values for coefficients of order 2 and 4 (Wagner 1972; Merson 1973). Coefficients of order 14, 13 and 12 are also likely to be best determined by resonance analysis.

For the comprehensive determination of the gravitational field, with, say, 300 coefficients (assuming truncation at degree and order near 16), the recipe is, first, a very large number of accurate observations (preferably several hundred thousand) of 20 or 30 satellites (at different inclinations, to avoid bias), from 20 or 30 observing stations well spread geographically. The next requirement is a sophisticated computer program to express the perturbations of satellite orbits moving in the force field defined by this 300-coefficient potential, with appropriate additions to allow for lunisolar perturbations, etc. Then further programs, and a very large and fast computer, are needed to solve, by least-squares methods, the observational equations, perhaps 500 000 of them, for the 300 geopotential coefficients, and perhaps 100 station coordinates. The computation is a marathon effort involving the inversion of immense matrices, but it has been done, and figure 4 shows the Smithsonian Standard Earth 1969, obtained from 100 000 photographic observations, mainly from the Baker–Nunn cameras, plus 3000 laser observations, which were at that time not much more accurate than photography (Gaposchkin & Lambeck 1971).

Figure 4 gives the contours of the geoid, or sea-level surface, relative to the best-fitting spheroid, which has a flattening of 1 part in 298.25. There is a depression south of India, 113 m deep, and a hump 81 m high near New Guinea. This means that a boat sailing along the equator south of India to north of New Guinea (with diversions to avoid land) would increase its distance from the Earth's centre by about 190 m during the voyage, although it would never go 'uphill'. The other main humps in sea level are about 60 m high, concentrated near Britain and south of Madagascar; and the other major depressions, southeast of New Zealand and off California and Florida, are about 50 m deep. The general shape of the geoid given by this map can now be regarded as well established; and it is worth remembering, because it is just as significant as the shape of the continents, being the outward expression of irregularities within the Earth. To remember the shape, look at the dark areas, and you will see that the western hemisphere is presided over by an animal rather like a goat, which appears to be in conversation with a man from the east, who dominates Asia.

Figure 4 may be seen as the final flower of the photographic era in satellite geodesy. Since 1970, when this map was published, improved geoid models utilizing laser observations of satellites and extensive analysis of gravity survey measurements have been produced at the Goddard Space Flight Center (Lerch, Wagner, Richardson & Brownd 1974), at the Smithsonian Astrophysical Observatory (Gaposchkin 1974), and also by the U.S. Navy Research Group, using Doppler

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observations (Anderle 1974). But to discuss these would be to go beyond the prologue. The map shown in figure 4, accurate to about 5 m over much of the world, is the basic terrain across which the researchers armed with the new techniques are marching forward.

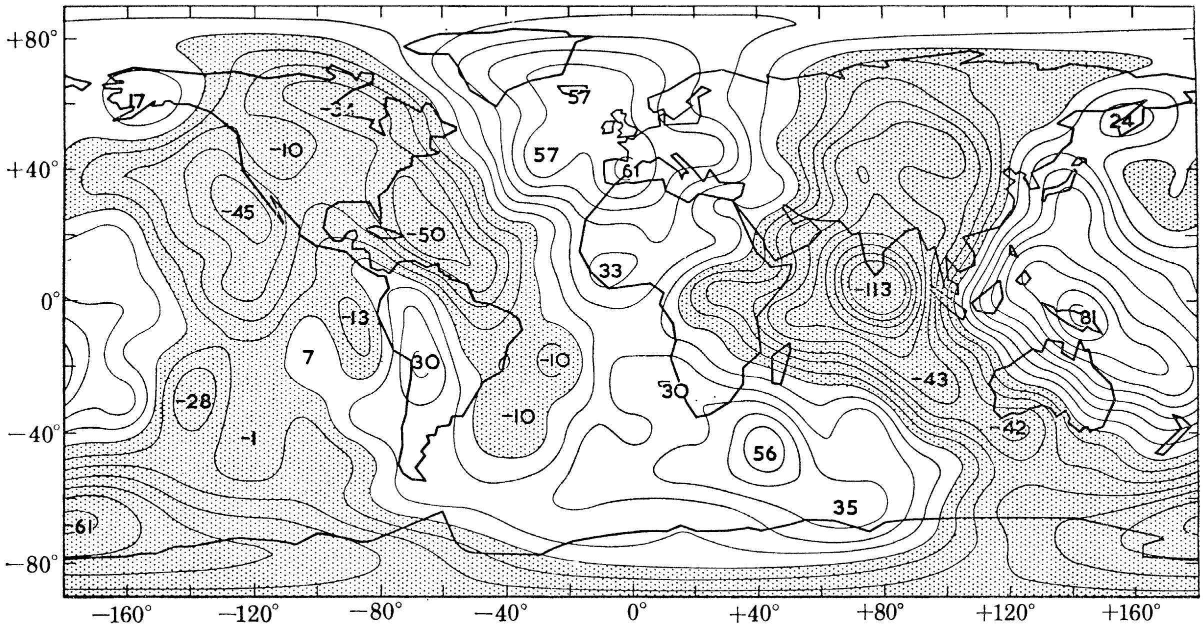

FIGURE 4. Geoid of the Smithsonian Standard Earth II. Contours at 10 m intervals, relative to a spheroid of flattening 1/298.25.

Lunar studies

While satellite laser ranging offers a straight road ahead from a secure base, laser ranging to reflectors on the Moon is more like 'a great leap forward'. The planting of reflectors on the Moon has some resemblances to the launching of artificial satellites of the Earth: it brought about as fundamental an improvement in accuracy; and the reflectors have, in a sense, brought the Moon into the family of artificial satellites, by allowing observations of an accuracy comparable with those of artificial satellites, and, as a corollary, requiring orbital theory of comparable accuracy.

The theory of the lunar orbital motion originates with Isaac Newton, and was subsequently improved by many famous mathematicians, including Clairaut, Euler and Laplace. An excellent historical summary is given by E. W. Brown (1896). The lunar theory developed by Brown (1897–1908) was the basis for calculating the position of the Moon until the 1960s, though contributions to lunar theory have been made by many other devoted workers in celestial mechanics: for a review see Brouwer & Clemence (1961). Before the planting of the corner-cube reflectors on the Moon, the theory had been developed to the point where its accuracy was compatible with the best observations, by radar and occultation methods, which were accurate to not better than a few hundred metres. Even in the 1960s the theory did not include the J_3 harmonic in the geopotential, which causes a long-term oscillation of about 3 km in the perigee distance of the Moon. The prospect of laser ranging accurate to 10 cm was in effect a demand for calculations at least 1000 times more accurate than were currently made. The attempts to achieve these improved accuracies in the past few years have relied on numerical integration and inclusion of many extra small perturbations. However, new analytical approaches are being made in lunar orbital and libration theory, as described in later papers in this volume.

The classical theories of the lunar orbit treated the Moon as a point mass, but in reality the Moon has a complex gravitational field, which perturbs any satellites in orbit about it. During the 1960s the lunar gravitational field was determined by analysis of the Doppler observations of satellites in orbit about the Moon. The analysis was conducted partly on the assumption that the lunar gravitational field could be expanded in harmonics, as with the Earth; but the accelerations deduced from the Doppler measurements showed local anomalies, which suggested concentrations of mass in particular areas. The concept of 'mascons', concentrations of mass beneath certain lunar maria, has proved most fruitful, and has led to good knowledge of the lunar gravitational field (Sjogren 1976), though some doubt remains about the far side of the Moon. Combined with photogrammetry, particularly from the Apollo flights, the gravity data gives some geometrical information, but there are ambiguities and uncertainties, which provided one of the spurs for the lunar laser ranging.

There are some advantages in studying the Moon – or the Earth – in isolation; but we always have to remember that the Earth and Moon are inextricably linked in one dynamical system. The evolution of the Earth–Moon system is a difficult dynamical problem, first successfully analysed by G. H. Darwin (1880), who identified many of the processes involved, and particularly the slowing of the Earth's rotation rate by tidal friction and the consequent increase in the Moon's distance (see Jeffreys 1959). The problem is one of appalling complexity, involving the axes and rates of rotation of both Earth and Moon, the pattern and amplitude of the Earth and ocean tides, and the orbital dynamics, including solar perturbations, and the effects of tides raised by the Sun. There were few measurements which could throw light on these problems: one of the most successful was the study of the acceleration of the Moon by analysis of observations in historic times, or from ancient eclipse records (Newton 1970). The accurate measurement of distance from observatories on Earth to the retro-reflectors placed on the Moon should provide the geometrical information essential for a better understanding of the motion of the Earth–Moon system, and will be valuable in complementing the geophysical results obtained from ranging to artificial satellites, particularly on polar motion, Earth rotation and tectonic plate motions.

Finally, it is worth remembering that the applications of laser ranging in space may prove to be wider than implied in the papers in this volume. For example, lasers aboard a satellite may be used for ranging to reflectors on Earth; satellite-to-satellite ranging may prove feasible; and space probes, or satellites beyond the Moon's orbit, may be fitted with retro-reflectors.

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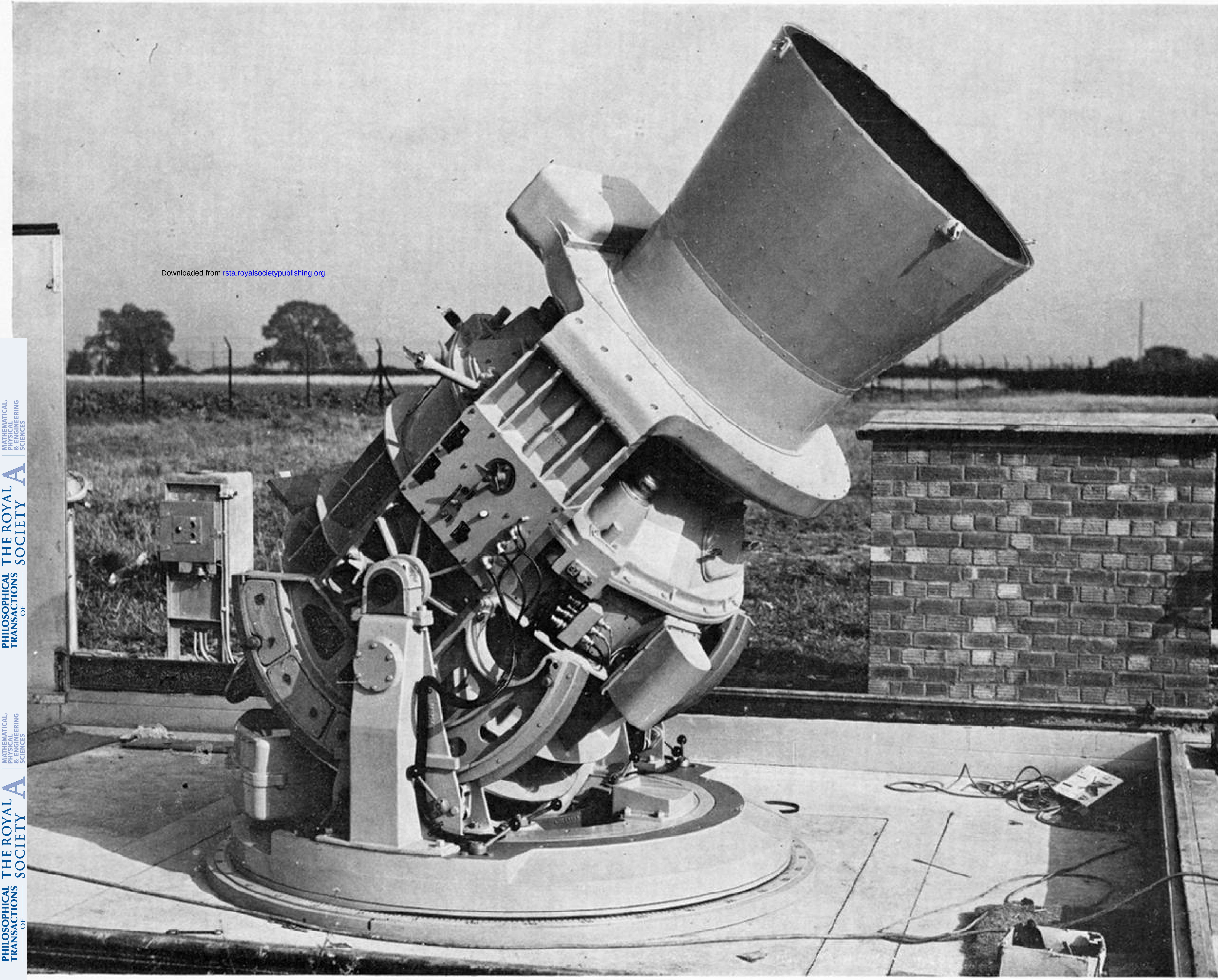

FIGURE 1. The 600 mm Hewitt camera operated by the Ordnance Survey at Malvern.

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